**Indian Statistical Institute, Bangalore**
**B. Math (II), First semester 2017-2018**
**Mid-Semester Examination : Statistics (I)**

**Date: 14-09-2017** **Maximum Score 40** **Duration: 3 Hours**

1. Consider a sequence of *binomial distributions Bin(n,$\theta_n$), $n \geq 1$.* Let $\theta_n \to 0$ as $n \to \infty$ *such that* $n\theta_n \to \lambda > 0$ as $n \to \infty$. Let the mean and variance of *Bin(n,$\theta_n$)* be denoted respectively by $\mu_n$ and $\sigma_n^2$. Find $\lim_{n\to\infty} \mu_n$ and $\lim_{n\to\infty} \sigma_n^2$. Also obtain limit of *pmf* of *binomial distribution, Bin(n,$\theta_n$),* as $n \to \infty$ and identify the limiting distribution. $[3 + 7 = 10]$

2. Let $X_1,\ X_2, \cdots, X_n$ be a random sample from the distribution whose probability density function (*pdf*) is proportional to

$$g\left(x|\theta\right) = \left(x-\theta\right)^{a-1}\left(1+\theta-x\right)^{b-1} I_{(\theta,\theta+1)}\left(x\right)\ ; -\infty < \theta < \infty \text{ unknown and } a, b > 0 \text{ known.}$$

Obtain $E\left[\left(X_1\right)^r\right]$. Find *method of moments (MOM) estimator* for $\theta$. Find *maximum likelihood estimator (MLE)* for $\theta$. $[3 + 3 + 5 = 11]$

3. A discrete model that is often used for the waiting time $X$ to failure of an item is given by the *pmf*

$$f\left(k|\theta\right) = \theta^{(k-1)}(1-\theta); k = 1, 2, \cdots;\ 0 < \theta < 1.$$

Suppose that we only record the time of failure, if the failure occurs on or before $r$ and otherwise just note that the item has lived at least $r+1$ periods. Let $Y$ denote this censored waiting time. Write down the *pmf* of $Y$. If $Y_1, Y_2, ..., Y_n$ is a random sample from this censored waiting time distribution, obtain *method of moments (MOM) estimator* for $\theta$. $[3 + 6 = 9]$

4. Consider the following data set

127, 144, 93, 184, 79, 53, 195, 57, 202, 80, 95, 161, 204, 108, 124
67, 102, 310, 177, 89, 146, 110, 141, 94, 118, 69, 63, 100, 207, 160

   (a) Make stem and leaf plots of these data.

   (b) Find the sample mean $\overline{X}$.

   (c) Find $100p$-th percentile for $p = 0{\cdot}75$.

   (d) Find the median $M$ and the third quartile $Q_3$.

   (e) Draw the box plot and identify the outliers.

   (f) Explain how to obtain the trimmed mean $\overline{X}_{\text{T}}$. Decide on trimming fraction just enough to eliminate the outliers and obtain the trimmed median $M_{\text{T}}$.

   (g) Explain (need not compute) how to obtain the trimmed standard deviation $S_{\text{T}}$.

   (h) Between the box plot and the stem and leaf plots what do they tell us about the above data set? In very general terms what can you say about the population from which the data have arrived? $[3 + 2 + 2 + 2 + 3 + 3 + 2 + 3 = 20]$